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Notes on Indian Astronomy

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I

THE LIBRATION OF EQUINOXES

In the Suryya Siddhanta and other Hindu Astronomical works, we find the theory of the Libration of Equinoxes in which it is stated that the first point of Aries moves along the Ecliptic twenty-seven degrees on each side of the Nirayana Vindu, the fixed initial point. That is, in a certain number of years it goes twenty seven degrees away from the Initial point, then returns-to it, again goes twenty-seven degrees the other side and comes back to the Initial point in a certain number of years.

The following extract from Brennand's Hindu Astronomy will show the universal acceptance of the theory of the Libration of Equinoxes and the Solstices by the Hindu astronomers. "This was a doctrine of a Libration of the Equinoctial and Solstitial points. Colebrooke in his essay on the equinoxes, has given the views of a number of writers on the subject; by some the motion is considered to be an entire revolution, through the whole of the asterisms; by others, and those the most numerous it was a libration, between certain limits on each side of a fixed point:.....In these two statements it may be noticed that Bhascara supposes the Equinoctial point is in motion, whereas the Suryya Siddhanta assumes that the entire circle of the Asterisms oscillates, first 27° on each

side of a mean point and then 27 degrees on the other side of that point. This supposed motion of the whole of the constellations may have led Bentley to assume that the ancient astronomers had two systems of Lunar Asterisms.....The theory of a libration, as expressed in various astronomical works, has been shown by Colebrooke to have been generally prevalent from very early times. It was also a doctrine maintained by Aryabhatta and Parasara, and by most of the Hindu astronomers of later times" (pp. 77-79). But this theory has been refuted by modern European astronomers[1] as will be seen from the following quotation from Lokmanya B. G. Tilak's Orion, page 82. "The hypothesis is now given up by modern astronomers as mathematically incorrect; but no reason has yet been assigned why it found place in the Hindu astronomy. A theory may be erroneous but even an erroneous theory cannot become prevalent without a good cause. It has been suggested by Bentley and approved by Prof. Whitney, that the limits of the libration might have been determined by the fact that the earliest recorded Hindu year had been made to begin when the sun entered the asterism of Krittika or 26° 40′ in front of Revati. But this alone is not enough to suggest the theory of libration. For, unless the Hindu astronomer had grounds—to him conclusive and otherwise inexplicable—for holding that the vernal equinox fell 27° on each side of Revati, he would not have proposed the libration of the equinoxes. So far as I know no such grounds have yet been discovered by modern scholars....." In this paper an attempt has been made to mathematically establish the conclusive grounds on which the Hindu astronomers based their theory of libration of Equinoxes.

[1] Vide also Burgess's translation of the Suryya Siddhanta, pp. 244 to 249.

Now looking at the curve of the Equation of Time herewith appended, we find that the Equation of Time is influenced by two factors—the Obliquity of the Ecliptic and the Eccentricity. The curve for the Equation of Time due to the Obliquity and that due to the Eccentricity are drawn separately. Let us start for convenience from the time when the Aphelion coincided with the first point of Aries (which happened about 1000 B. C.)[1] and let us also assume that the Eccentricity and the Obliquity do not vary during one complete revolution. At this time the Equation of Time is zero at the vernal equinoctial point. As years pass on the aphelion goes ahead of the first point of Aries at the rate of 62 seconds (50·2″ for precession and 11·8″ for the movement of the apsides) annually. Now looking at the curves we find that the maximum equation of time is $\pm 7\frac{3}{4}$ minutes due to the Eccentricity and this is attained about 90 degrees after the Perihelion or the Aphelion point (more correctly 88° 50′ after the Perihelion point). The corresponding degree in the curve for the obliquity at which the equation is $\pm 7\frac{3}{4}$ minutes is 27 degrees about, on either side of the Equinoctial or the Solstitial points.

This may also be seen from the following solution :—

The maximum Equation of Time due to Eccentricity being 7·68 minutes (1° 55′) and that due to Obliquity being 9·9 minutes (2° 28′), the mean Longitude of the point at which the Equation of Time is 7·68 min., on the curve for the Obliquity will be given by solving the following :

In a spherical right-angled triangle in which the hypotenuse is L, the mean longitude (26° 30′). the angle adjacent is Ω, the Obliquity of the ecliptic (23° 27′)

[1] The earth being in aphelion on the vernal equinoctial day about 4000 B.C., graphically, first point of Aries and aphelion coincided then.

and the base is the R. A. (right ascension) to be found, we have

$$\text{Tan R. A.} = \frac{\text{R cos } \Omega}{\text{cot L}}, \text{ (R = radius).}$$

$$= \frac{\text{R cos } 23°27'}{\text{cot } 26°30'}$$

R = 10.
cos 23° 27′ = 9·9626
cot 26° 30′ = 10·3023

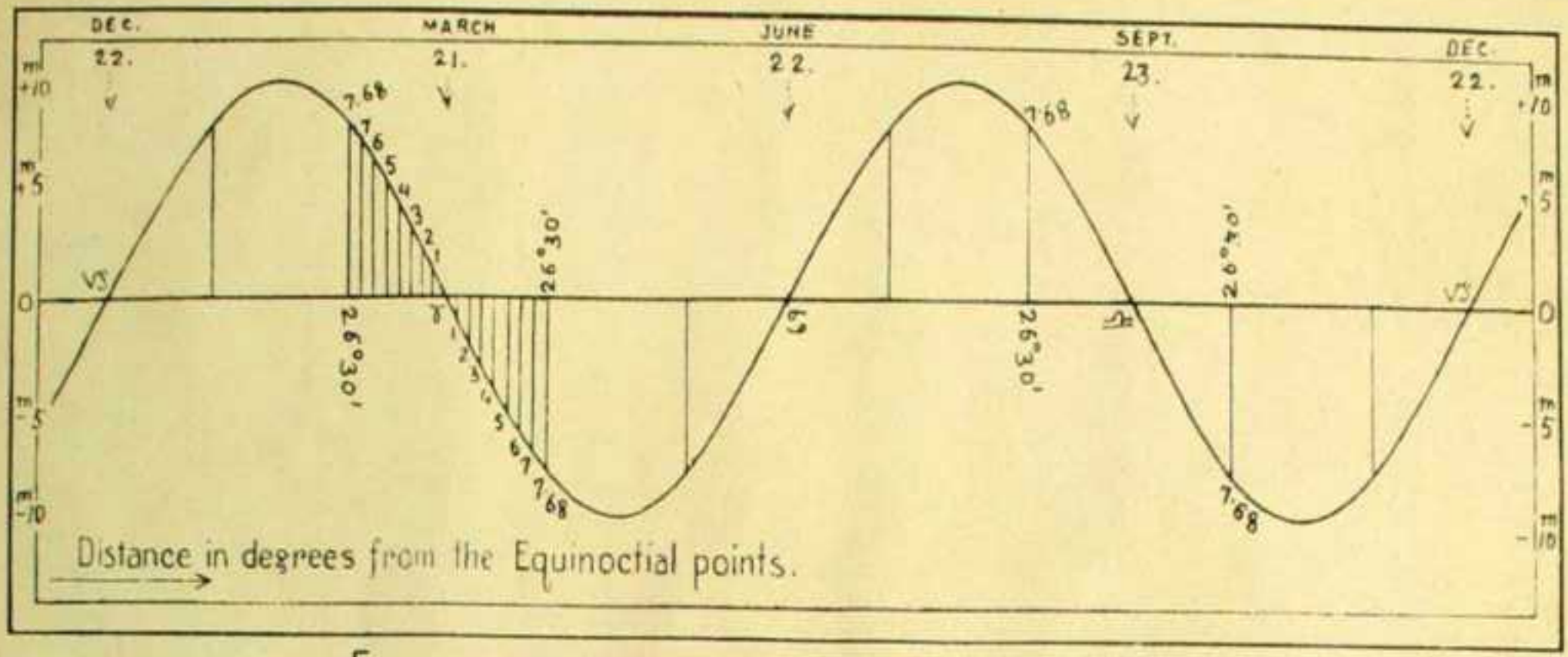
Tan R. A. = 9·6603
R. A. = 24° 35′

∴ Equation of time due to Obliquity = L − R. A. = 26° 30′ − 24° 35′ = 1° 55′ (7· 68 min.).

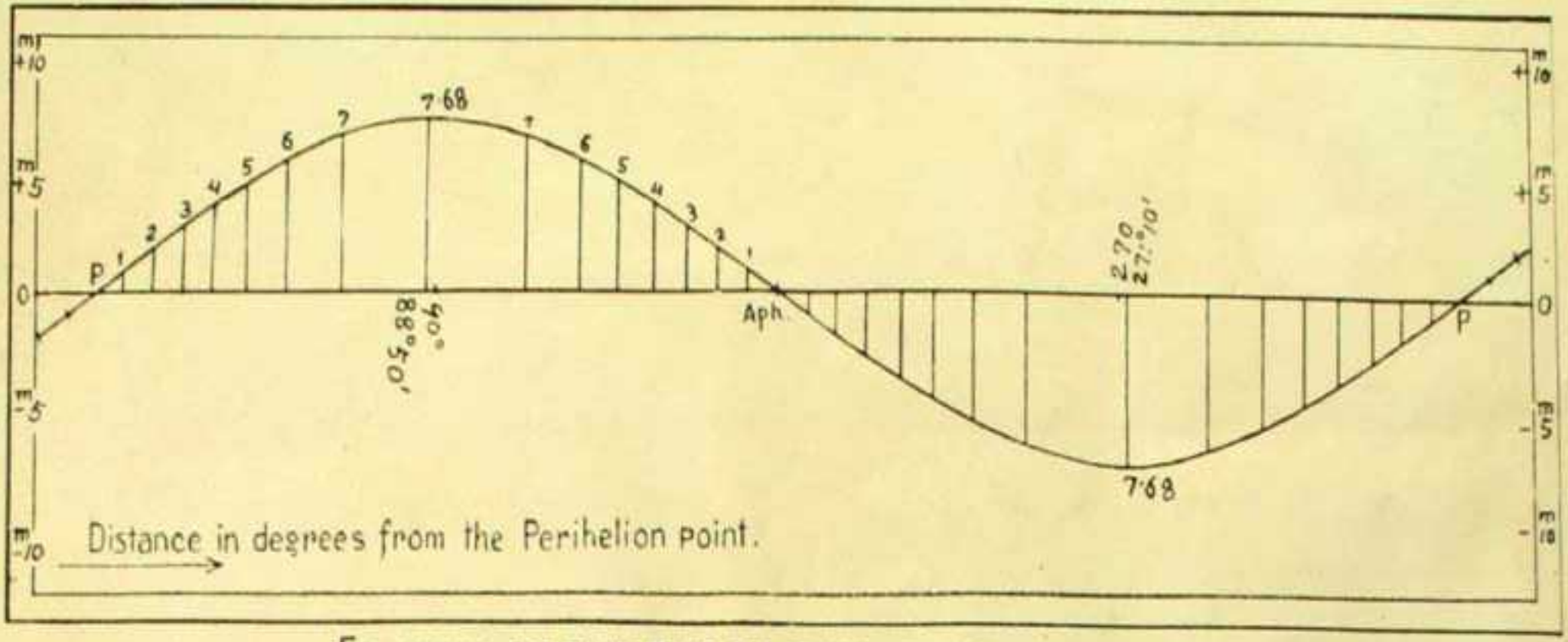
∴ the mean longitude of the point at which the equation of time is 7·68 min. on the curve for the Obliquity is 26° 30′.

So the point near Aries at which the Equation of Time is zero oscillates about 27 degrees on either side of it.

Therefore when the Aphelion has advanced (90 plus 27) or 117 degrees from the first of Aries the longitude of the Sun on the day at which the Equation of Time is zero, is 27 degrees. This to occur, we have to shift the curve for the Eccentricity about 120 degrees keeping the curve for the Obliquity fixed. When the Aphelion will be advancing still further the Equation of Time will be zero—on dates earlier until the perihelion coincides with the first of Aries, at which time the Equation is zero at the vernal equinoctial day. This to take place the Eccentricity curve has to be shifted 60 degrees more. Let the aphelion advance 60 degrees further. The Equation is zero again at a point 27 degrees on the other side of the first of Aries after which time it is zero on earlier days and the aphelion advancing 120 degrees more coincides again with the first of Aries, when the equation is zero.



EQUATION OF TIME DUE TO OBLIQUITY OF ECLIPTIC.



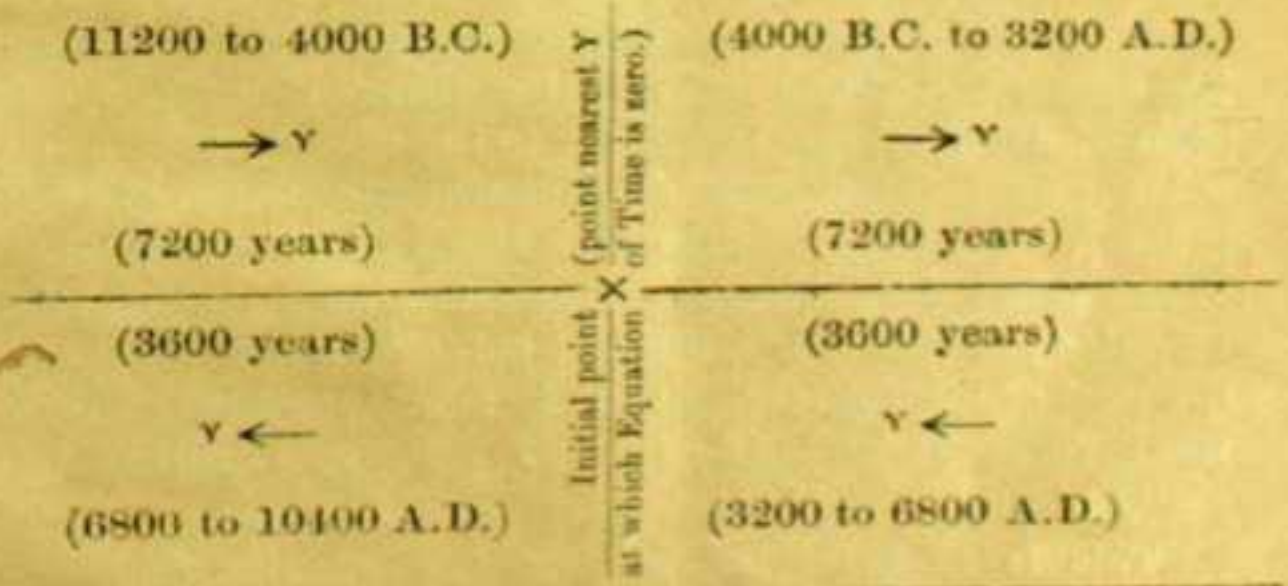
EQUATION OF TIME DUE TO ECCENTRICITY.

Now assuming the point near Aries at which the Equation of Time is zero as the fixed Initial point it will be seen very clearly that the first point of Aries moves 27 degrees on one side or the other of this Initial point. This solution may similarly be extended to the other Equinoctial or the Solstitial points.[1]

The periods of the above oscillations are the following :

1st period (120 degrees movement) $= \frac{120^\circ}{60^\circ} = 7200$ years.
2nd ,, (60 ,, ,,) $= \frac{60^\circ}{60^\circ} = 3600$,,
3rd ,, (60 ,, ,,) $= \frac{60^\circ}{60^\circ} = 3600$,,
4th ,, (120 ,, ,,) $= \frac{120^\circ}{60^\circ} = 7200$,,

Total 21600 years. Double this amount equals 43200 years. The reason for the adoption of the period of a Yuga as 432 and so many zeros is now apparent. The oscillatory period of 108 degrees is 21600 years (about 20 86 according to modern European Astronomy). Therefore in a Yuga of 4320000 years there are 200 complete revolutions. The period of 21600 years may be divided into three periods of 7200, 7200 and 7200 (3600 plus 3600) years. Each of these periods is the 600th part of a Yuga.



[1] The maximum Libration of Solstices is 24° 30'. This explains Aryabhatta's [illegible] of the Libration of Solstices (Ayana) as 24 degrees.

This is the reason why Ranganath the great astronomer commentator of the Suryya Siddhanta mentions this revolution as *bilakshana*, possessed of peculiar characteristics.

It is clear, the Hindus carried their observation assiduously at least from 12000 B.C. to about 3500 B.C. to expound the Libration of Equinoxes in which case the Hindu civilization is at least 14000 years old.

Now to find the Ayanamsa. When the R A. of the Sun is 120 degrees his longitude is about 117 degrees. When the distance between the Nirayana Vindu and aphelion is 90 degrees, the distance between the first of Aries and the Nirayana point is 27 degrees; this is the Ayanamsa then. In other words the difference between the longitude of the sun on the day on which the equation is zero nearest the vernal Equinox, and that of the vernal Equinox is the Ayanamsa for a particular year. Taking the case of the present time the difference between the longitude of the sun on the 14th of April and when the earth is in aphelion on July 2nd (which is the *bhuja*), about 77 degrees, the Ayanamsa should be $\frac{77 \times 27}{90} = 23$ degrees about. This is the same as the longitude of the Sun on the 14th of April at which date the equation is zero.

We also observe that when the mean time is less than the apparent time on the vernal equinoctial day, the first point of Aries is to be looked for to the east of the Initial point (the point nearest Aries at which equation of time is zero). This was the case from about 13000 to 4000 B.C. When mean time is greater than the apparent on the vernal equinoctial day, the first point of Aries is towards the west of the Initial point, as it is at present. This is what verse 11 and 12—Chap. III, Suryya Siddhanta—really mean.

This is in brief the explanation of the Libration of Equinoxes as expounded by the ancient Hindu

Astronomers. The Hindu Nirayana year is neither Sidereal nor Tropical. It is a new thing altogether. This is an addition to one of the clearest arguments of the independent origin of Indian Astronomy.

Taking the first day of the Hindu Nirayana year to indicate the day when the Equation of Time is zero nearest the Vernal Equinox (about April 15th now) it is clear that Vernal Equinox cannot happen earlier or later by 27 days, in a cycle of about 21600 years.

If now the study of Hindu Astronomy and chronological events be conducted in the light of the above explanation, I am confident all anomalies and seeming absurdities will disappear to the great joy of the astronomers.

With great diffidence, being a young student of astronomy,—I place these few lines before the mathematicians and astronomers—with the hope that they will examine the above statements and see if the Libration Theory of the Hindus has been correctly solved.

In conclusion, I acknowledge my gratefulness to Asutosh Mittra, Esq., Professor, Vidyasagar College, from whom I got valuable help in understanding difficult portions of the Suryya Siddhanta, but for whose help this explanation might not have struck my imagination.

Addendum (to paper on Libration of Equinoxes).

Burgess in his translation of the Suryya Siddhanta (Journal of the American Oriental Society, Vol. VI) pages 244 to 249, discussed the theory of Libration of Equinoxes expounded by the ancient Hindu astronomers. Here I quote a few lines from his remarks :—

"..... Now it is not a little difficult to suppose that a phenomenon of so much consequence as this, which enters as an element into so many astronomical processes

should.....have been hidden away thus in a pair of verses....." p. 246.

".....Besides these considerations drawn from the general history of Hindu astronomy, and the position of the element of the precession in the Suryya Siddhanta we have still to urge the blind and incoherent, as well as unusual form of statement of the phenomenon, as fully exposed above....." p. 247.

".....Bhaskara's own commentators.....hold to that of a libration, which has been and is altogether the prevailing doctrine throughout India and seems to have made its way thence into the Arabian and even into the early European astronomy (see Colebrooke, as above)." What Bhaskara mentions in his Siddhanta Siromani (Goladhyaya-Golabandhadhikara-slokas 17 and 18) as 'Ayana Chalanam' (movement of the first point of Aries or Libra) and the period of revolution is clear from his own notes on the same. This is nothing but the movement of a fixed equinox with reference to the aphelion. The period of a complete revolution of the perihelion or the aphelion with reference to a fixed equinox is according to modern astronomy 20986 years (annual movement 61·9″ seconds). This according to Bhaskara is 21636 years (annual movement 59·9″ seconds). This is the same as the period of one complete libratory movement.

Now I shall make it clear that the Greeks also adopted the same sort of year calculation as the Hindus. Young in his "Manual of Astronomy," page 144, says, "He (Hipparchus) found that the year of the seasons, from solstice to solstice, as determined by the Gnomon, was shorter than that determined by the heliacal rising and setting of the stars (*i.e.*, the times when certain constellation rise and set with the sun), just as if the Equinox preceded, *i.e.*, 'stepped forward' a little to meet the sun." Regarding the amount of precession determined by

Hipparchus, Burgess says, "Among the Greek astronomers Hipparchus is regarded as the first who discovered the precession of Equinoxes; their rate of motion, however, seems not to have been confidently determined by him, although he pronounces it to be at any rate not less than 36″ yearly."

".....Ptolemy however was so unfortunate as to adopt for the true rate Hipparchus's minimum of 36″ a year ....." p. 249. Now in the explanation of the Libration of Equinoxes, I have shown that in about 7200 years (from B.C. 4000 and onwards) the maximum libration of about 27° will be attained (please see the last diagram in the paper). Now assuming the Sun's daily motion to be 1°, we see that in 7200 tropical years and 27 days, 7200 Nirayana years are completed. Therefore one Nirayana year is $\frac{27}{7200}$ day or 0·00375 day longer (under the present conditions) than the tropical year. Therefore the present Nirayana year is equal to (365·24219+·00375) days or 365·24594 days. The difference between the Sidereal and tropical year being 0·01417 day, during this time 50·1″ seconds of arc of the ecliptic are passed over. The difference between the Nirayana year (for the present) and the Sidereal year being 0·01042 d., the number of seconds of arc passed over during this time is got by simple proportion:

$$0{\cdot}01417 \text{ d} : 0{\cdot}01042 \text{ d} :: 50{\cdot}1'' : x''$$

$$\therefore \quad x = \frac{{\cdot}01042 \times 50{\cdot}1}{{\cdot}01417} \text{ or } 36{\cdot}8''.$$

This is the amount of precession determined by Hipparchus and Ptolemy. Therefore they certainly used this year which began on the day when the mean time and Sun dial time were the same nearest the Vernal Equinox. This to be true the length of the year calculated by Hipparchus and Ptolemy ought to be about 0·00375 d. or 5·34 minutes longer than the tropical

year[1] and I find this is to be exactly the case, "Professor Newcomb who has compiled an instructive table of the Equinoxes severally observed by Hipparchus and Ptolemy, with their errors deduced from Leverrier's Solar Tables, finds palpable evidence that the discrepancies between the two series were artificially reconciled on the basis of a year 6 minutes too long, adopted by Ptolemy on trust from his predecessors. He nevertheless holds the process to have been one that implied no fraudulent intentions."—(Encyclopædia Britannica, ed. 1910, p. 810, Astronomy). It is clear what Hipparchus and Ptolemy meant was the movement of the first point of Aries (the fixed point in the ecliptic indicating the beginning of the constellation Aries) with respect to the point where the said year ended. Now I leave it to the judgment of the astronomers to pronounce if the remarks "The rate of motion seems not to have been confidently determined by him (Hipparchus)—or—Ptolemy was so unfortunate as to adopt for the true rate Hipparchus's minimum of 36″ a year....." are justified. The Hindus have still retained this mode of year beginning.

The theory of the Libration of Equinoxes was prevalent also in Arabia for a considerable time, having found its way into Arabian and early European Astronomy from India (*vide* Colebrooke): as such the following

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[1] On modern data vernal equinox and perihelion coincided about 4089 B. C. (Smyth's Cycle of Celestial Objects) and now the equation of time is zero nearest the vernal equinox on April 16. That is in 4089 plus 1921 or 6010 years a difference of 25 days (from 22nd March to 16th April) has happened. That is in round numbers, in 6000 years the increase is 25 days. Therefore one year $= \frac{25}{6000}$ d. $= \frac{25 \times 24 \times 60}{6000}$ min., or 6 minutes longer than the tropical year. It is clear the Nirayana year of the Hindus is the apparent solar year. This type of year will continue to be longer than the tropical till the maximum libration is attained, after which it will lag behind or continue being shorter than the tropical till maximum libration on the other side of the vernal equinox will be attained. However, we notice that the length of 20986 (modern data) apparent solar years is the same for 20986 mean solar (tropical) years.

lines from Smyth's 'Cycle' will be read with keen interest by all. "Thabit (Thabit-Ibn-korrah) was also a clever astronomer and is said to have improved what Ferghani prepared: he, however, is principally remarkable for having revived the old notion of a variation in the position of the ecliptic, as well as in the fixed stars, which has been called the trepidation: It was his opinion founded on some erroneous observations(?), that the stars moved for some time according to the order of the signs; that they afterwards proceeded in a retrograde direction, and returned to their former places, after which they assumed a direct motion; and that they then had an irregular motion, which was rapid for a certain period, then became slower, and at last insensible. He maintained that the obliquity of the ecliptic was variable under similar periods of increase and decrease; and his opinions prevailed for a considerable time,"—p. 28. A glance at the figure in page 281, will explain the above statements regarding 'trepidation' or Libration of Equinoxes. Therefore now the explanation of the theory of the Libration of Equinoxes will be evident to all.

II

THE LIBRATION OF COLURES AND CONSEQUENT CHANGES IN THE COMMENCEMENT OF THE HINDU NIRAYANA YEAR

In my last note I endeavoured to interpret in the correct way the theory of Libration of Equinoxes universally advocated by the Hindu astronomers. The Libration theory being accepted the next problem which presents itself, is, how to keep the relation between the fixed asterisms (Rasis and Nakshatras) and the months of the year (inseparable with the Hindu astronomers), unchanged, when both the first point of Aries and the point nearest it where the Equation of Time is zero, are not fixed with respect to the stars. From the following considerations it will be seen how this problem had been solved in the past by the Hindu astronomers and how it has to be tackled in future.

The point in the Ecliptic nearest the Vernal Equinox at which the Equation of Time is zero, marks the starting point for the commencement of the Hindu Nirayana year (Nirayana from Nir and ayana, no parting of the mean and apparent Suns about the Vernal Equinoctial point). When this point recedes 20 degrees from a point in the Ecliptic which is recognised as the beginning of one of the twelve constellations of the zodiac (the fixed Rasis), a change is introduced in the beginning of the year, namely one solar intercalary month is added to keep the relation between the fixed asterisms and the names of the months intact. In so doing the precessional period (the period of the revolution of the colures) has to be

taken into account, as will be seen from the following example :

Assuming the Vernal Equinox and the Equation of Time zero, coincident in 4000 B. C, at 79 degrees Longitude (Hindu data), they were again coincident 10800 years before that time. During this time Vernal Equinox receded 10800 × 48″ seconds=144°. Therefore at Longitude 223° (79° plus 144°) the Equation of Time was zero and the Vernal Equinox was also there. On modern data this happened when the Vernal Equinox was happening at about 29° Longitude.

Brennand in his Hindu astronomy 1 page 77, says "To the theory of the revolution of the colures there was a rival doctrine..... This was the doctrine of the Libration of the Equinoctial and the Solstitial points." These are however no rival doctrines, but theories going hand in hand with the Hindu astronomers. The period of the revolution of colures is, in round numbers 27000 years according to the Hindu astronomers (25868 years in modern astronomy). If now the whole Ecliptic be divided into equal portions each with a precessional period of 1000 years, we get 27 such portions (each representing 13° 20′). This is the origin of the Hindu system of the twenty-seven Nakshatras—twenty-eight with Abhijit (alpha Lyrae, Vega) the brightest star in the northern hemisphere occupying the remarkable situation of the Pole Star of the ancient period when the month of Agrahayana was the first month of the year. (Abhijit from prefix 'abhi' and 'ji' जि to conquer.) Abhijit became the pole star and thus being the conqueror, all heavenly bodies began to circle round Abhijit as if in obeisance. When Abhijit was no longer the pole star, naturally it was left out of the reckoning.

The table appended herewith has been constructed to show the probable periods of the successive year

This table is constructed with the following data—Magha=Regulus (Longitude 148°), at the ninth degree of Rashi Sinha (Constellation Leo). Hindu period of Libration—21600 years (Modern 20986 yrs.); Precessional period—27000 years (Modern 25868 years).

Maximum of Libration=27°

Month beginning the period.	Date.	Periods begun.		Duration of Period (Hindu)	Eq. of time zero at (or year begins at).	Asterisms coinciding.	Asterism 180° behind the former.	Vernal Equinox.		Asterisms coinciding.	Asterisms at Autumnal Equinox.	Pole Star.
		Modern.	Hindu.					Modern.	Hindu.			
Agrahayana.	1st	15905 B.C.	16528 B.C.	yrs. 1425	259°	Mula.	Mrigasira.	248·1°	246°	Jyestha. (Antares). Abhijit. (Lyra).	Rohini. (Aldebaran).	Abhijit. (Vega).
Kartika	„	14519 „	15103 „	1455	229°	Bisakha.	Krittika.	228·8°	227°	Bisakha.	Krittika.	Do.
Aswina	„	13132 „	13648 „	1432	199°	Chitra.	Aswini.	209·5°	207·6°	Swati.	Aswini.	Do.
Bhadra	„	11640 „	12216 „	1958	169°	U. Phalguni.	P. Bhadrapada.	190°	188·5°	Hasta. Apas (δ Virginis).	Revati.	...
Sravana	„	9647 „	10258 „	3105	139°	Magha.	Dhanistha. (Srabistha).	161°	162·4°	P. Phalguni. (δ Leonis.)	P. Bhadrapada, Satbhisaj. (λ Aquarius.)	...
Ashada	„	6629 „	7153 „	3153	109°	Punarvasu. (Castor Pollux).	U. Ashada.	119°	121°	Pushya. Procyon.	Sravana.	...
Jyaistha	„	3683 „	4000 „	3125	79°	Mrigasira.	Mula.	78°	79°	Mrigasira. β Tauri. Aurigæ.	Mula.	α Draconis.
Baisakha	„	665 „	875 „	3130	49°	Krittika.	Bisakha.	36°	37°	Bharani.	Swati. (Arcturus).	...
Chaitra	„	2281 A. D.	2255 A. D.	1954	19°	Aswini.	Chitra.	355°	355·6°	U. Bhadrapada.	U. Phalguni. (δ. Leonis.)	α Ursæ Minoris. (Polaris).
Phalguna	„	4977 „	4209 „	1425	349°	P. Bhadrapada.	U. Phalguni.	330°	329·5°	Satabhisa.	Magha. (Regulus).	
Magha	„	5442 „	5634 „		319°	Dhanistha.	Magha.	311°	310·5°	ε, γ. Aquarius. Sravana.	Aslesha.	...

beginnings, the names and dates of the months beginning the year for the periods, etc., etc. This is computed according to ancient Indian and modern data, starting from Agrahayana as the first month of the year. The difference between the two sets of results is not much owing to the fact that the ratios between the two sets of data are about the same.

$$\frac{27000}{21600} = 1{\cdot}25\ ;\ \text{also}\ \frac{25868}{20986} = 1{\cdot}233.$$

On looking at the table for the period beginning with the month of Agrahayana we notice that the year began when the sun was exactly in the beginning of the asterism Mula. The Vernal Equinoctial colure passed through Jyestha (Antares) and Abhijit (Lyra),[1] the star Abhijit (Vega) was also the pole star of the period. The autumnal Equinox passed through Rohini (Aldebaran), and six months after the first of Agrahayana we have the Orion group (Mrigasiras). The solstitial colure passed through Purva Phalguni (delta Leonis) and Satabhisaj (λ Aquarius). Now the naming of the month as Margasirsa and the significance of "I am Margasirsa of the months and Abhijit of the Nakshatras" of the Bhagabata is now clear. The origin of the naming of Nakshatra Mula (the root), Jyestha (the first) and of Rohini from *arohan* or

[1] At the commencement of the period Abhijit (Vega) was about 8 degrees behind the equinoctial colure. About 200 years later Abhijit was about 5 degrees behind the same. This is the time to which the 'Aitareya Brahmana' refers as the Abhijit day falling four days before the Vishuvan (Equinox). The late Mr. Tilak was not precise in his statement that with Punarvasu (Pollux) at the vernal equinox Abhijit would be about 6 degrees behind the autumnal equinox (Orion, p. 202). With the vernal equinox in Punarvasu, Abhijit is at least 30 degrees in front of the autumnal one. Mr. Tilak overlooked the fact that the longitudes given in the Suryya Siddhanta are polar. The time when Abhijit was of importance as found in the oldest Vedic literature, and the time when the importance of Abhijit was gone (as in the Taittiriya Samhita) are now clearly discernible.

abarohan, ascent or descent of the Sun from the Equator, according as the observer is situated to the north or the south of it, will now be understood.

Next coming to the period when the year began with the month of Kartica we meet with the peculiar phenomenon that here the Equation of Time was zero at the Vernal Equinoctial point (at about 229°, the Longitude of β and 20 Librae). According to the Hindu data when the Vernal Equinox was at 223°, the Equation of Time was zero exactly at that point and here is the star alpha Librae, Bisakha, exactly on the Ecliptic. Pictorially α Librae represents the pointer, β and 20 Librae the two arms of the balance. Thus a balancing of the two things have occurred in the beginning of this period and the origin of the naming of the Nakshatra Bisakha (from *dvisakha*) the two arms of the balance and that of the Rasi Tula (Libra from libration) from *tulana* indicating equipoise will now be evident. A similar junction of the two phenomena occurred at 82° Long. (modern data), or 79° Long. (Hindu data), and here is the beginning of the Rasi Mithuna (Gemini): and thus the origin of the name of the Rasi Mithuna is clear.

Now when the point at which the Equation of Time is zero—nearest the Vernal Equinox will coincide with the Longitude of the star Revati, a change has to be introduced. The exact year at which this will happen has to be determined by practical astronomers. In a congress of the Indian astronomers it has to be declared that that particular year will have two Phalguna months and thenceforward the 1st of Chaitra will be the first day of the succeeding years until another change has to be effected, which will not take place before about 1800 years after that. For this change the Hindus will not have any objection seeing their ancestors have done the same several times—(*vide* Tilak's Orion, pp. 198-220).

At the commencement of the period beginning with the month of Baisakha, Vernal Equinox fell at 36 degrees and the Equation of Time was zero in the asterism of Krittika. At this time the Nakshatra Krittika was the first of the series :—

"देवग्रहा वै नक्षत्राणि।···कृत्तिका: प्रथम°। विशाखे ह्युत्तम°। तानि देव नक्षत्राणि।·····" तैत्तिरीय ब्राह्मण।

But during Varaha Mihira's time Vernal Equinox was happening in Revati and the Equation of Time was zero, that is, the year began, in Aswini (alpha Arietis). This being so Varaha Mihira and others introduced the Aswini system in place of the Krittikas. In case of the moon we are allowing one intercalary lunar month in every three years and we shall have to do the same in the case of the sun, allowing him one intercalary solar month at the end of, say, 2000 or 3000 years, to keep his relation with the Rasis and Nakshatras intact.

Here I have tried only to give a general survey of the Hindu system of year beginnings consistent with and consequent on the theory of the Libration of Equinoxes which should no longer be considered as a mere matter of speculation in which the Hindus revelled, but an astronomical truth which they discovered after assiduous observations extending over thousands of years.

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III

THE SO-CALLED SIDEREAL YEAR AND THE SIDEREAL PERIODS OF THE PLANETS IN HINDU ASTRONOMY.

The term Saura Varsa (Solar year) is met with in all Hindu Astronomical works. This is translated as the Sidereal year, the period for which is given in the Suryya Siddhanta and other Hindu Astronomical works. Now why should the Saura (Solar) year be understood as the Sidereal year, I am at a loss to understand. Burgess in his translation of the Suryya Siddhanta, page 409,—remarks "—The Solar Year as already noticed is sidereal not tropical;—The length of the solar year and month is subject only to an infinitesimal variation, due to the slow motion of 1 minute in 517 years, assumed for the Sun's line of apsides;—" Now the question is if the Hindu Solar Year is sidereal how can the length of it vary at all even though it be after hundreds of years; I have never heard of any astronomer saying that the sidereal year is variable because of the movement of the Sun's line of apsides. It is to be noted that the sidereal year estimated upon an average sufficiently large, possesses the essential quality of a standard unit, that of complete invariability. Evidently, because it was wrongly assumed that the motion of the apsides is only 0·1161" seconds per annum, it was conjectured that this was nothing but the Sidereal Year, the difference being infinitesimal (2·79 seconds). "The period of the planets' revolution about the centre of the epicycle is the time which it takes the latter to make the circuit of the orbit from the apsis

around to the apsis again, or the period of its anomalistic revolution. This is almost precisely equal to the period of sidereal revolution in the case of all the planets excepting the moon, since their apsides are regarded by the Hindus as stationary."—Burgess' Translation of S.S., page 207. But the right course should have been to put it that the Hindu Solar Year is anomalistic, and to say, that with the Hindu astronomers the difference between the sidereal and the anomalistic year is negligible. Then it would have been easier for one to find if the motion of the Sun's line of apsides is correct as given in the Siddhantas. The fact is that the annual motion of the apsides should be 11·61″ seconds and not 0·1161″ seconds. The number of revolutions of the Sun's apsis in a Kalpa is given in the S. S. (Suryya Siddhanta) as 387. This is certainly the number of revolutions in 432×10^5 years. The number of revolutions calculated from modern astronomy is 393. The revolutions in a Kalpa of 432×10^7 years should be 38700. The annual movement of the line of apsides in modern astronomy is 11·79″ seconds. (Godfray has it, 11·25″) Consequently the anomalistic year is 365 d. 6 h. 13 m. 48 s. (365·25958 mean solar days). The Hindu solar year is 365 d. 6 h. 12 m. 36·6 s. (365·25876 days), the difference being only one minute and eleven seconds. This year was certainly not meant by the Hindu astronomers to be used as the year of chronology and civil reckoning. That part was left to the Nirayana year to accomplish. Of course it started from an epoch in which the Equinox, apsis and the fixed star which indicated the beginning of the first zodiacal sign, coincided. It was then observed after what interval of time two or all of these phenomena coincided. As an example let us assume that Vernal Equinox and perihelion coincide after 21600 years, the Vernal Equinox and the particular star coincide after 27000 years. Therefore after five

coincidences of the Equinox and the apsis (5 × 21600 years or 108000 years) or four coincidences of the Equinox and the star (4 × 27000 years) or 108000 years, the Equinox, apsis and the particular star coincide. The period in which mankind in general are interested is the tropical year, on which the return of the seasons depends. But in this tropical year no account is taken of the position of the apsides on which the—severity or mildness of the seasons depends. The Hindu Nirayana year is the resultant period of the movement of the Equinox and the apsis (please see my first article on "Libration of Equinoxes") and as such it is the period in which mankind in general are most interested. Now-a-days perihelion being in January and aphelion in July, we northerners, are having short, less severe winter and long mild summer. But after ten thousand years from now when aphelion will be in January and perihelion in June, the extreme summer and winter of the southern hemisphere will be transferred to us. Therefore the adoption of the Nirayana Year by the Hindus is more natural and scientific than the tropical. The period of the Saura Varsa (Solar Year—anomalistic) was mentioned in the Siddhantas as it would be of much help to the astronomers for their calculations.

Now I shall pass on to the so-called sidereal periods of the planets, Mercury, Venus, Mars, Jupiter and Saturn as found in Hindu astronomical works. The period of Mars is given in the S. S. as 686·99749 mean solar days. The sidereal period in modern astronomy is 686·9797 days. The mean annual movement of the apsis of Mars is 15·82 seconds. Therefore the apsis of Mars has advanced 15·82 × 1·88 (sidereal period of Mars in years) seconds in one sidereal revolution. On calculation the anomalistic period of Mars comes out to be 686·9902 days.

The period of Saturn as given in the S. S. is 10765·77

days. The sidereal period of Saturn in modern astronomy is 10759·22 days, a difference not to be easily passed over. Now the mean annual movement of Saturn's apsides is 19·37 seconds. Therefore the apsis of Saturn has moved in one sidereal revolution 19·37 × 29·46 (period of Saturn in years) seconds or 570·64 seconds. The mean daily motion of Saturn being 120·5 seconds it will take (570·64 ÷ 120·5) or 4·74 days more for Saturn to reach the perihelion. Therefore the anomalistic period of Saturn comes out to be 10763·96 days. But it is worth while noticing that the period as corrected by the *Bija* is 10764·89 days. Similarly the sidereal period of Mercury being 87·9693 days, the anomalistic period is 87·9694 days (assuming the mean annual movement of the apsis to be 5·84 seconds). The period in the Suryya Siddhanta is 87·9697 days. The sidereal period of Venus being 224·7008 days, and the mean annual movement of her apsis being 2·68 seconds towards the west, one would naturally expect the period as given in the Siddhantas to be shorter than the sidereal period (supposing the Hindu periods anomalistic), and strange enough the period in the Suryya Siddhanta is 224·6986 days, that corrected by the *Vija* is 224·6990 mean solar days (the anomalistic period calculated on modern data is 224·7005), both shorter than the sidereal period. The sidereal period of Jupiter being 4332·58 days, the anomalistic period should be 4332·85 days (assuming the mean annual movement of the apsis to be 6·65 seconds). The period in the Suryya Siddhanta is 4332·32 days. This even after correction by the *Vija* is 4332·42 days, shorter than the sidereal period.

This shortening of the period of Jupiter and the lengthening of that of Saturn are due to this: "The remarkable fact, however, that the mean motion of Jupiter was then more rapid and that of Saturn less so than it had formerly been, was detected. This

anomalous phenomenon, which is now so well known to be caused by the mutual perturbations of those planets on each other, was a startling difficulty; but Cassini investigated the conditions, and boldly conjectured that the time would arrive, when those effects would be of a contrary nature. His happy prediction has been beautifully verified."—Smyth's 'Cycle,' p. 52.

Now one may ask, assuming these periods in the Siddhantas to be anomalistic what about the period of the moon which is exactly sidereal? The answer to this is that the position of the full-moon among the well known twenty-seven asterisms of the Hindus gives rise to the names of the months and this is happening since time immemorial, and as such astronomers are bound to give prominence to her sidereal period. "Naturally enough since the moon is the most conspicuous of the nightly luminaries, and her revolutions more rapid and far more important than those of the others, the asterisms would practically be brought into much more frequent use in connexion with her movements;..."—Burgess: Translation of the Suryya Siddhanta, page 352. The anomalistic and draconitic (nodical) periods may be easily calculated from the data given in the Siddhantas.

Here I add a table of the revolutions and movements of the apsides and nodes of the Planets according to the Suryya Siddhanta and modern data. The revolutions are for the period 432×10^5 years, assuming the values as given in Watson's Theoretical Astronomy and in Smyth's Cycle of Celestial Objects to remain constant through ages. The revolutions according to the Suryya Siddhanta are certainly for the same period (432×10^5 years). The design in putting the revolutions of the Planets in 432×10^5 years is that we can find the movement in a century in seconds by simply multiplying by 3. Thus the revolution of the Sun's apsis in 432×10^5 years being 387,

Planets.	Revolutions in 432×10^4 years. (Modern)	(S. S.)	Movement in a Century. (Modern)	(S. S.)
Mars				
apsis	527	204	1582"	612"
node	776	214 (207-Bhaskara)	2329"	642" (801"-Bhaskara)
Mercury				
apsis	195	368	584"	1104"
node	261	488 (521-Bhaskara)	782"	1464" (1563"-Bhaskara)
Jupiter				
apsis	222	900	665"	2700" *
node	527	174	1581"	522"
Venus				
apsis	89	535 (653-Bhaskara)	268"	1605" (1959"-Bhaskara)
node	624	903 (893-Bhaskara)	1871"	2709" (2679"-Bhaskara)
Saturn				
apsis	646	39	1937"	117"
node	647	662	1942"	1986"
Sun				
apsis	393	387	1179"	1161"
Moon				
apsis		4882030	401" (daily)	14646090" (400"·98-daily)
node		2322380	190"·6 (daily)	6967140" (190"·74-daily)

* Maraldi the gifted nephew of Cassini, by recurring to an observation made B. C. 300, and comparing this with the position during his time, calculated that in 1934 years Jupiter's node have retrograded with respect to the fixed stars, as much as 14½ degrees. This gives the movement in a century as 2709 seconds.

the movement in a century is (387×3) or 1161″ seconds. Hence the annual movement is 11·61″ seconds. In some cases it will be seen that the values are almost the same with modern mean values, in others certainly errors have crept in. A glance at the table will show that the values for Mars and Mercury had been interchanged. From the manner of writing the two words Kuja, कुज Budha बुध or Kauja (कौज) and Baudha (बौध) in Devanagri type this error has crept in. The same error will be observed about Mars and Mercury while speaking of the retrogradations of the Planets in verses 53 and 54, chapter II, Suryya Siddhanta. In support of this as to how errors have crept in and accumulated, I shall quote here what that great genius Bhaskaracharyya said in connection with this very subject in his Siddhanta-Siromani :

"सा तु तत्तत् भाषा-कुशलैन तत्तत् शिक्षकज्ञानज्ञन युतशीलैन एव शीतुं शक्यते न अन्येन। यदस्मद् शीशोच पाता: स्व स्व मार्गेतु गच्छन्त एतावत: पर्य्यवान् कल्पे कुर्वन्ति इति यत आगम एव प्रमाणम्। स च आगमो महता कालेन लेखक अध्यापक पर्य्येक्षदोषै: बहुधा जात:। तदा कतमश्च प्रमाणम्। अथ यदि एवम् उच्यते गणितस्कन्ध उपपत्तिमानेव आगम: प्रमाणम्। उपपत्त्या ये सिध्यन्ति भगणा: ते याह्या: तदपि न। यतो अतिप्राज्ञेन पुरुषेण उपपत्ति: ज्ञातुमेव शक्यते। न तथा तेषां भगणानाम् इयत्ता कर्तुम् शक्यते। पुरुषायुषाऽल्पत्वात्। उपपत्तीतु यत्न: प्रत्यहं यत्नेन विध्य:। भगणानां यावत्। एवं ग्रहेश्वरस्य तावत् वर्षाणां सिद्धता भगण: पूर्य्यते। मन्दोच्चानां तुवर्षशतैर् अनेकै:। अतो नायम् अर्थ: पुरुष साध्य: इति। अतएव अतिप्राज्ञा गणका: साम्प्रतीयलब्धि अनुसारिण प्रौढगणक लोकृत कमपि आगमम् अङ्गीकृत्य यद्गणिते आकाशे गणित गोलयो: निरतिशय कौशल दर्शयितु तथा अन्यै: भान्तिज्ञानेन अन्यथा उदितान् अर्थांश निराकर्तुम् अन्यान् ग्रन्थान् रचयन्ति। यद्गणिते इतिकर्त्तव्यतायाम् अस्माभि: कौशले दर्शनीय भवतु आगमो योऽपि कोऽपि अयम् आशय: तेषाम्। यथा अत्र यन्त्रे ब्रह्मगुप्तस्वीकृत आगमो अङ्गीकृत इति। तर्हि तिष्ठतु तावत् उपपन्ना भगणानाम् इयत्ता साधनम्। अथ यदि उपपत्ति: उच्यते तर्हि इतरेतराश्रयदोष शतथा वक्तुम् अशक्या। "

ग्रहगणित मध्यमाधिकारे भगणाध्याय: ।

Now some may doubt that as the revolution is said to be completed when the Planet returns to Revati, how can these be the anomalistic periods ? I wish them to ponder over the original text : " .....पौष्णान्ते भगण: स्मृत:।"

पौष from पूषन् (the sun)[1] अन्ते निकटे (पौषस्य रेवती योग-तारायां अन्ते निकटे प्रदेशे, etc., Ranganath), whether it does not mean the perihelion and consequently the anomalistic period. The secondary meaning is the star Revati with which the apsis was coincident in the beginning. Similarly 'Meshadi' (first of Aries) or end of Revati came to mean the Initial or starting point—the first point of Aries in the case of tropical revolution the aphelion point in the case of anomalistic revolution, etc. *Vide Surya Siddhanta* verses 45, 48, 57 and 67, Chap. XII, and Ranganath's commentary on verse 48. "मेषादौ विषुवद् वृत्तस्थ क्रान्तिवृत्तभागे रेवत्यामग्रे, etc." Moreover the mention of Revati 'Tara' does not mean that we are to look always for a particular star shining in the heavens just as we are not to understand the expression 'Dhruva Tara' as the two pole stars shining through ages in the heavens, but simply as the poles: 'ध्रुवयोर्दक्षिणोत्तरस्थिरतारयो वर्ह ब्रह्मणानिवर्ह, etc.'—Ranganath, notes on sloka 73, Chap. XII, also verse 43 of the same chapter मेरोरुभयतो मध्ये ध्रुवतारे नभःस्थिते। निरक्ष देश संस्थानामुभये क्षितिजाश्रये॥ "In both direction from Meru are two pole-stars, fixed in the midst of the sky: to those who are situated in place of no latitude, both these have their places in the horizon" Regarding 'Bhagana'—द्वादशराशिभोगाद् भगण इत्यर्थः—Ranganath, note on verse 27, Chap. I. These rasis may be Sayana, Nirayana, etc.—"द्वादशराश्यङ्किते वृत्ते उच्चस्थानात् चतुर्विभागात्मक एकैको भागो राशिचयात्मकोपदसंज्ञः—Ranganath, Chap. II. 29. *Vide* also "अजादि केन्द्रे सर्व्वेषां गेहे मान्दे च कर्म्मणि। धनं ग्रहाणां लिप्तादितुलादाद्यणमेव च॥—Chap. II. 45. "There is nowhere in this work any allusion to them (Hindu names of the signs) as constellation, or as having any fixed position of their own in the heavens: they are simply the names of the successive signs (rasi, bha) into which any circle is divided, and it is left to be determined

[1] Synonyms for रेवती—"अन्त्यर्क्षं रेवती पौष्णं पूषा इति च कथ्यते।"

by the connection, in any case, from what point they shall be counted."—Burgess, Translation of the S.S., page 181. Still more I should ask learned men to think over the meaning of Revati from रेङ्-प्लवगतौ प्लवगतिः प्लुतगतिः—Siddhanta Kaumudi (motion in very long interval of time): "एकमात्रो भवेद्ह्रस्वो द्विमात्रो दीर्घ उच्यते। त्रिमात्रस्तु प्लुतोज्ञेयो....."—याज्ञवल्क्य शिक्षा। (मात्रा—interval), and the design in the naming of the star will be evident to all.

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